# Multiscale 3-D texture segmentation: Isotropic Representations

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# Collaborators & Acknowledgements

This work has been performed in collaboration with Simon K. Alexander (UH), Robert Azencott (UH and ENS) and Manos Papadakis (UH).

We gratefully acknowledge the contributions and previous related work by B.G. Bodmann (UH), S. Baid (formerly UH), D.J. Kouri (UH), X. Li (UH) and Juan R. Romero (U.Puerto Rico).

# Outline

# Biomedical Imaging and segmentation

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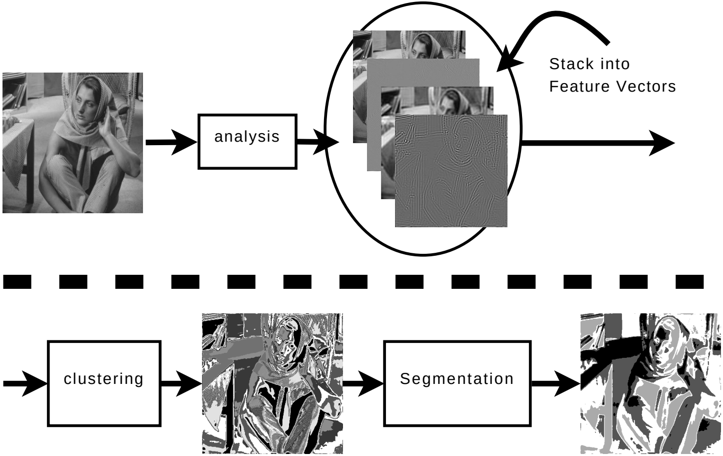
- This assists diagnostic imaging especially when latent tissues are picked up by this process.
- A robust algorithm must be immune to rigid motions of the image.
- We require a multi-scale structure because textures manifest themselves at various scales.

# Segmentation flowchart

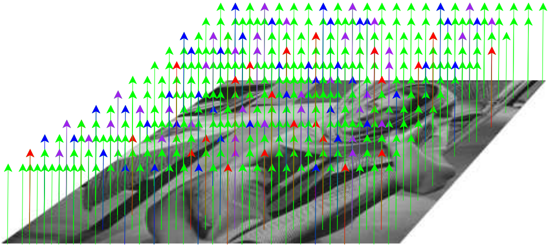


# Image Lattice

For all $z \in \mathbb{R}^d$ associated data $f(z)$

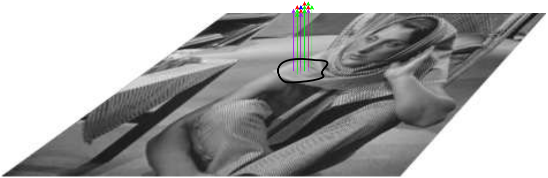
# Feature Vectors on Image Lattice

Associated feature vectors $Af(z)$. Each vector co-ordinate is a wavelet coefficient.



# Texture Patch

We care only about local Neighborhoods $\mathcal{N}(z)$ forming texture patches of related features



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- To each pixel $z \in \mathbb{R}^3$, we associate a 'feature' vector belonging to a fixed finite dimensional space $V$, called the feature vector space.
- The vector space of all $V$-valued functions $h$ defined on $\mathbb{R}^3$ such that $\|h(\cdot)\|$ is square integrable on $\mathbb{R}^3$ is denoted by $H$. Thus, $\|h\| = \left(\int_{\mathbb{R}^3} \|h(z)\|^2 dz\right)^{1/2}$.

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- Let $A : L^2(\mathbb{R}^3) \to H$ be a bounded linear transformation that associates to each $f$ the $V$-valued function $Af$ defined on $\mathbb{R}^3$. This linear transformation is called a feature map.

# Steerability

## Definition

Consider a bounded linear feature mapping $A$ generating for each image $f$ and each voxel $z \in \mathbb{R}^3$ a feature vector $Af(z)$ in the Euclidean space $V$. We shall say that $A$ is a *steerable feature mapping* if there is a mapping $U$ from the group $G$ of rigid motions into the general linear group $GL(V)$ of $V$ such that for each rigid motion $R$ of $\mathbb{R}^3$, the invertible transformation $U(R)$ verifies,

$$A[\mathcal{R}f](z) = U(R)\left[Af\left(R(z)\right)\right] \quad z \in \mathbb{R}^3.$$

Here $\mathcal{R}$ is the following transformation induced by $R$ on $L^2(\mathbb{R}^3)$:

$$\mathcal{R}f(z) = f(Rz).$$

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We summarize the properties that the feature vectors should have:

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- Good clustering of relevant textures.
- The features must provide localized information.
- A multi-scale structure to describe textures at different scales.

Hence, Multiresolution Analysis is an ideal choice.

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- Inflexible design due to the orthogonality constraint.
- The segmentation algorithm needs approximately 5-15 dimensional feature vector space. This is hard to achieve with DWT using applicable length scales.

# Problems with tensor product basis

Barb image

DB8 decomposition

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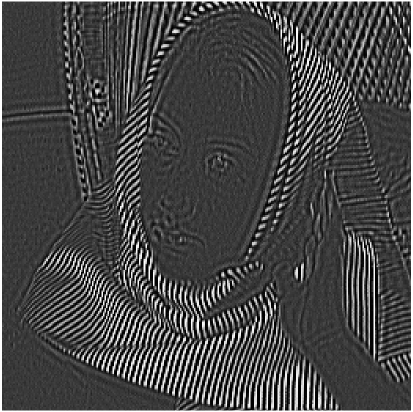
DB8 high pass

# Problems with tensor product basis

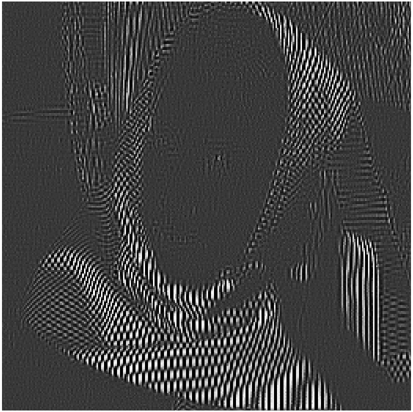
IMRA high pass

DB8 high pass

# Problems with tensor product basis (Zoom)



IMRA high pass



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- Bessel families
- efficient computation of features
- the decomposition to be isotropic (at least approximately) to avoid directional bias
- steerability

# Outline

# Definition

An IMRA is a sequence $\{V_j\}_{j\in\mathbb{Z}}$ of closed subspaces of $L^2(\mathbb{R}^n)$ satisfying the following conditions:

- $\forall j \in \mathbb{Z},\ V_j \subset V_{j+1}$,
- $(D)^j V_0 = V_j$,
- $\cup_{j\in\mathbb{Z}} V_j$ is dense in $L^2(\mathbb{R}^n)$,
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- $V_0$ is invariant under all rotations.

Then $V_0$ must be a Paley-Wiener space of a certain form, i.e. there exists a measurable set $\Omega$ invariant under all rotations such that

$$f \in V_0 \Leftrightarrow \hat{f}(\xi) = 0 \ \ \xi \notin \Omega.$$

# An example of IMRA

Let $\phi \in L^2(\mathbb{R}^d)$ a refinable function i.e. there exists $m_0 \in L^\infty(\mathbb{T}^d)$ such that

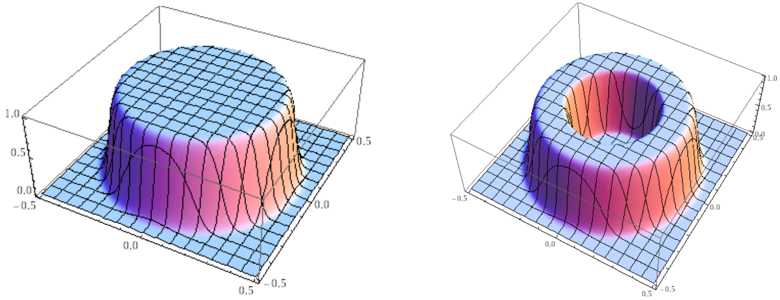
$$\hat{\phi}(2\cdot) = \hat{\phi} m_0.$$

We assume that $\hat{\phi}$ is smooth and satisfies the following:

- $\hat{\phi}$ vanishes outside the ball centered at the origin with radius $r < 1/2$
- $\hat{\phi}(\xi) = 1$ for all $\xi$ in the ball $B(0, \frac{1}{4})$

Define, $\psi$ via $\hat{\psi}(2\cdot) = \hat{\phi} - \hat{\phi}(2\cdot)$ .

# 2D IMRA scaling function and wavelet




(a) Fourier transform of the scaling function $\hat{\phi}$

(b) Fourier transform of the wavelet $\hat{\psi}$(2.)

Using the functions $\phi$ and $\psi$, we define the following feature map:

$$Af(z) = (< f,\ T_z\phi(\cdot/8) >,\ \ < f,\ T_z\psi(\cdot/4) >,\ \ < f,\ T_z\psi(\cdot/2) >)\,.$$

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- This map is both translation and rotation invariant
- Hence, it is steerable.

# Directional Selectivity

IMRA High Pass tensor $\sin(\theta)$

IMRA High Pass tensor $\cos(\theta)$

# Better than tensor product basis

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D8 High Pass

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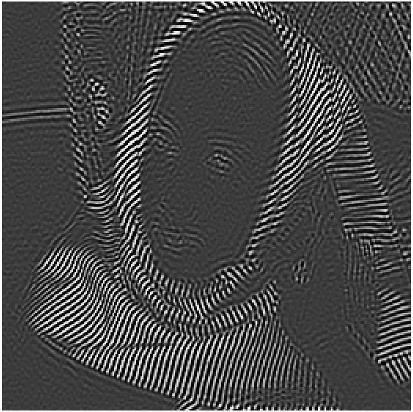
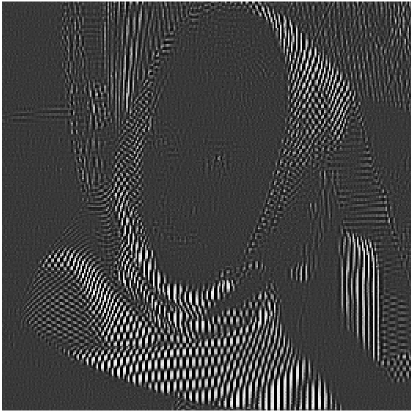
# Better than tensor product basis (Zoom)

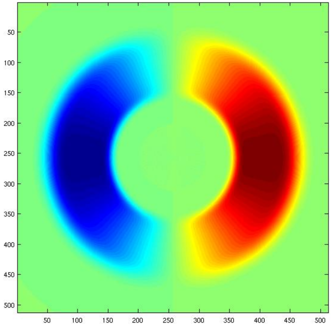


IMRA High Pass tensor $\cos(\theta)$

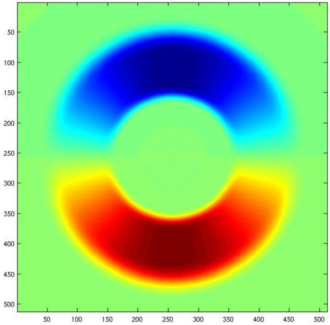


D8 High Pass

# The filters




(c) High Pass tensor $\sin(\theta)$




(d) High Pass tensor $\cos(\theta)$

# Outline

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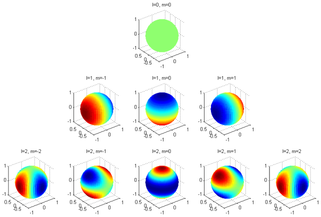
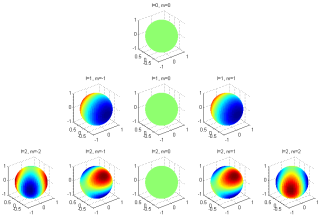
- $d_m$ is equal to 2 in the case of $\mathbb{R}^2$ or $2m + 1$ in the case of $\mathbb{R}^3$.
- For $m = 0$ we only have $n = 0$ and $Y_{0,0} = 1$.

# 3D Spherical Harmonics of degree 0, 1 and 2

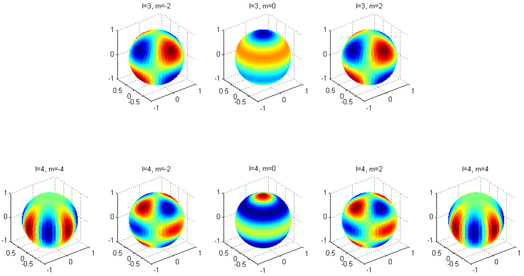



(e) Real part




(f) Imaginary part

# More 3D Spherical Harmonics




(g) Real part of 3D Spherical Harmonics of degree 3 and 4

# Thank you for your attention !!

# References

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