Proof of mean difference

- The probability that a dimer $D$ is synthesized during $[t, t+dt)$ is given by $a_2(\xi)\,dt$.

- The probability that a dimer $D$ is decomposed into two monomers $M$ during $[t, t+dt)$ is given by $a_3(\xi)\,dt$.

\[
a_2(\xi) = \frac{K_2}{2} (2 + Z_1 - Z_{10} - 2Z_2 + 2Z_3) (2 + Z_1 - Z_{10} - 2Z_2 + 2Z_3 - 1) \\
a_3(\xi) = K_3 (4 - 2Z_4 + 2Z_6 - 2Z_7 + 2Z_2 - Z_3)
\]

Equilibrium implies that $a_2(\xi)\,dt = a_3(\xi)\,dt$.

Therefore,

\[
\frac{K_2}{2} (2 + Z_1 - Z_{10} - 2Z_2 + 2Z_3) (2 + Z_1 - Z_{10} - 2Z_2 + 2Z_3 - 1) = K_3 (4 - 2Z_4 + 2Z_6 - 2Z_7 + 2Z_2 - Z_3)
\]

\[
(z_2 - z_3)^2 - A(z_5)(z_2 - z_3) + B(z_5) = 0
\]

where

\[
A(z_5) = 2 + Z_1 - Z_{10} + \frac{K_3}{2K_2} - \frac{1}{2}
\]

\[
B(z_5) = \frac{1}{4} (1 + Z_1 - Z_{10}) (2 + Z_1 - Z_{10}) - \frac{K_3}{2K_2} (4 - 2Z_4 + 2Z_6 - 2Z_7 + 2Z_2)
\]

The result

\[
\mu^2(2; j_1, \xi_3) - \mu^2(3; j_1, \xi_3) = \frac{1}{2} \left[ A(z_5) - \sqrt{A^2(z_5) - 4B(z_5)} \right]
\]

is obtained by solving the previous quadratic equation for $z_2 - z_3$ and by taking conditional expectations with respect to $z_5$. 

- The propensity function for reaction #2 \((2X_2 \rightarrow X_3)\) is given by
  \[
  \Pi_2(x_1, x_2, x_3, x_4, x_5, x_6) = k_2 \frac{x_2(x_2-1)}{2}
  \]

- The propensity function for reaction #3 \((X_3 \rightarrow 2X_2)\) is given by
  \[
  \Pi_3(x_1, x_2, x_3, x_4, x_5, x_6) = k_3 x_3
  \]

- Recall now that \(\overline{X}(t) = x_0 + S \Xi(t)\).

- In this example,
  \[
  x_0 = \begin{bmatrix}
  0 \\
  2 \\
  4 \\
  2 \\
  0 \\
  0
  \end{bmatrix}
  \]

- In addition, if you derive \(\delta\), you will find that
  \[
  x_2 = 2 + z_1 - z_0 - 2z_2 + 2z_3
  \]
  \[
  x_3 = 4 - z_4 + z_5 - z_6 + z_7 + z_2 - z_3
  \]

- Recall that \(\alpha_m(\Xi) = \Pi_m(x_0 + S \Xi)\).

- From \(\ast\), \(\ast\ast\), \(\ast\ast\ast\), and \(\ast\ast\ast\ast\) we obtain
  \[
  a_2(x) = \frac{k_2}{2} (2 + z_1 - z_0 - 2z_2 + 2z_3) (2 + z_1 - z_0 - 2z_2 + 2z_3 - 1)
  \]
  \[
  a_3(x) = k_3 (4 - z_4 + z_5 - z_6 + z_7 + z_2 - z_3)
  \]