• The probability of next reaction.

\[ P_t(\tau,m) : \text{the probability that, given we are at time } t, \]
\[ \text{the next reaction occurs at time } t+\tau \text{ and that this is the } m-\text{th reaction} \]

\[ P_t(\tau,m) = \text{the probability } P_t^0(2) \text{ that no reaction takes place during the time interval } [t,t+\tau] \]
\[ \text{multiplied by the conditional probability that the } m-\text{th reaction occurs during } [t+2,t+2+2\tau], \text{ given that no reaction occurs within } [t,t+2] \]
\[ \text{m-th reaction} \]
\[ t \quad 2t + 2 \quad t + 2\tau \]
\[ \text{no reaction} \]

\[ P_t(\tau,m) = \mathbb{P}_t[\text{no reaction occurs during } [t,t+2]] \]
\[ \times \mathbb{P}_t[\text{m-th reaction occurs during } [t+2,t+2+2\tau] | \text{no reaction occurs during } [t,t+2]\]

\[ P_t(\tau,m) = P_t^0(\tau) \times \int_{2\tau}^{\infty} \alpha_m(\tau) d\tau \]
Now, we have

\[ P_t^* (2) = \left[ P_t^* \left( \frac{2}{L} \right) \right]^L \]

probability that no reaction occurs within an infinitesimally small interval of length \( \frac{2}{L} \).

\[ P_t^* \left( \frac{2}{L} \right) = 1 - \sum_{m=1}^{M} \alpha_m (\Xi(t)) \frac{2}{L} \]

Therefore:

\[ P_t^* (t) = \lim_{L \to \infty} \left[ P_t^* \left( \frac{2}{L} \right) \right]^L = \lim_{L \to \infty} \left[ 1 - \frac{2}{L} \sum_{m=1}^{M} \alpha_m (\Xi(t)) \right]^L = \exp \left\{ -\frac{2}{L} \sum_{m=1}^{M} \alpha_m (\Xi(t)) \right\} \]

Hence

\[ P_t(\tau,m) = \alpha_m (\Xi(t)) d \tau \times \exp \left\{ -\frac{2}{L} \sum_{m=1}^{M} \alpha_m (\Xi(t)) \right\} \]