Solutions #5

Problem 1: Clearly, \( d_1(u, v) \geq 0 \), for every \( u, v \). We need to show that the function \( d_1(u, v) \) satisfies the following three properties: (a) \( d_1(u, v) = 0 \) if and only if \( u = v \), (b) \( d_1(u, v) = d_1(v, u) \), and (c) \( d_1(u, w) \leq d_1(u, v) + d_1(v, w) \). Notice that:

\[
d_1(u, v) = |u_1 - v_1| + |u_2 - v_2| = 0 \iff u_1 = v_1 \text{ and } u_2 = v_2 \iff u = v,
\]

which shows (a). Also,

\[
d_1(u, v) = |u_1 - v_1| + |u_2 - v_2| = |v_2 - u_2| + |v_1 - u_1| = d_1(v, u),
\]

which shows (b). Finally,

\[
d_1(u, w) = |u_1 - w_1| + |u_2 - w_2| \\
= |u_1 - v_1 + v_1 - w_1| + |u_2 - v_2 + v_2 - w_2| \\
\leq (|u_1 - v_1| + |v_1 - w_1|) + (|u_2 - v_2| + |v_2 - w_2|) \\
= d_1(u, v) + d_1(v, w),
\]

which shows (c).

Problem 2: Given a shape \( F \), the translation invariant erosion of \( F \) by a disk structuring element \( rB \) of radius \( r \) (\( B \) denotes a disk of radius 1) is given by:

\[
F \ominus rB = \{u \in R^2 | rB + u \subseteq F\} = \{u \in R^2 | d(u, F^c) \geq r\},
\]

where \( d(u, F^c) \) denotes the distance of a point \( u \) from \( F^c \), given by

\[
d(u, F^c) = \begin{cases} 
\min\{d(u, v), v \in F^c\}, & \text{if } u \in F \\
0, & \text{otherwise}.
\end{cases}
\]

Notice that \( d(u, F^c) = D_F(u) \), where \( D_F(u) \) is the distance transform of \( F \). Therefore, a point \( u \in F \ominus rB \) if and only if \( D_F(u) \geq r \). This suggests that the erosion \( F \ominus rB \) is the result of thresholding \( D_F(u) \) at level \( r \).
Problem 3: Clearly, for the Euclidean distance, we have that:

This leads to: \( d_2(A, B) = \sqrt{3^2 + 2^2} = \sqrt{13} = 3.6056 \). On the other hand, for the geodesic distance, we have that:

Which leads to: \( d_F(A, B) = d_2(A, C) + d_2(C, B) = \sqrt{2^2 + 1^2} + \sqrt{1^2 + 1^2} = \sqrt{5} + \sqrt{2} = 3.6503 \). Clearly, 

\( 3.6056 = d_2(A, B) \leq d_2(A, C) + d_2(C, B) = d_F(A, B) = 3.6503 \).
Problem 4: We will apply the binary watershed transform by using the distance transform. First, the distance transform $D_F(u)$ is calculated and is inverted, in order to obtain $-D_F(u)$. The local minima of $-D_F(u)$ are then used to determine the required catchment basins (which are four of them) for the watershed transform. By flooding the catchment basins and by building dams to avoid water from a catchment basin to spill over an adjacent catchment basin, produces the watershed lines depicted in red below.
Problem 5: This problem is very similar to the problem of detecting the lean meat region in a beef stake image of Lecture 15. First, we threshold the grayscale image $f$ in order to obtain a binary image $F_{thr}$ that contains the large rectangle. Then, we erode $F_{thr}$ by a small structuring element $B_{\text{small}}$ and we subtract the result from $F_{thr}$. This gives us the external marker $M_{\text{ext}}$. Then, we erode $F_{thr}$ with a large structuring element $B_{\text{large}}$ in order to obtain a marker for the small rectangle. This gives us the internal marker $M_{\text{int}}$. The union of the external and internal markers gives us a combined marker $M = M_{\text{int}} \cup M_{\text{ext}}$. This marker $M$ marks the catchment basins on the grayscale data $f$. The watershed transformation produces the red line depicted in the figure below.
Problem 6: The segmentation problem here is rather simple. The grayscale image $f$ is thresholded in order to obtain a binary image $F_{\text{thr}}$ that contains the objects of interest. This image is inverted, and the SKIZ is applied on the resulting image $F_{\text{thr}}^c$. The SKIZ lines are depicted in red in the figure below.