Homework #4

1. Consider the following grayscale signal $f$:

   ![Grayscale Signal](image1.png)

   (a) Determine its threshold decomposition $\mathcal{F}$.
   (b) Is the stacking property satisfied?
   (c) Erode each cross section in $\mathcal{F}$ by the flat structuring element $B = \{-1,0,+1\}$ and calculate the grayscale function $f'$ after stacking the results.
   (d) Calculate the flat grayscale erosion $f'' = f \ominus B$. How does $f'$ compare to $f''$?

2. Consider the following two grayscale signals $f_1$ and $f_2$:

   ![Grayscale Signals](image2.png)

   (a) Calculate the signal $f_1 \lor f_2$ using only set-unions.
   (b) Repeat part (a) for calculating the signal $f_1 \land f_2$. 


3. Show the following four properties:
(a) \( f \ominus B \leq f \), if \( B \) contains the origin.
(b) \( f \ominus b_1 \geq f \ominus b_2 \), if \( b_1 \leq b_2 \).
(c) \( (f \ominus b_1) \ominus b_2 = f \ominus (b_1 \oplus b_2) \).
(d) \( f \oplus b = (f^* \ominus b)^* \).

4. Show that, if \( \{ \psi_i \} \) is a collection of closings, then \( \bigwedge \psi_i \) is a closing as well.

5. Let \( \mathcal{F} = \{ F(t) | t \in Z, F(t) \neq \emptyset \} \) be the threshold decomposition of a grayscale image \( f \) and \( \Psi \) be an increasing set operator. Set \( g(x, y) = \psi(f)(x, y) = \max \{ t \in Z | (x, y) \in \Psi(F(t)) \} \). Show that:
(a) If \( \Psi \) is a dilation, then \( \psi \) is a dilation as well.
(b) If \( \Psi \) is idempotent, then \( \psi \) is idempotent as well.
(c) If \( \Psi \) is anti-extensive, then \( \psi \) is anti-extensive as well.
(d) If \( \Psi \) is an opening, then \( \psi \) is an opening as well.

6. An image \( f \) is corrupted with max-noise \( \eta \). Let \( P_{f:B}(k) \) and \( P_{\eta:B}(k) \) be the pattern spectra of the image and noise, respectively, based on a structuring element \( B \). After extensive experimentation, it has been determined that the optimal smoothing filter (in the sense of minimizing the expected absolute difference metric) is given by
\[
\psi(g) = (g - g\ominus3B) + (g\ominus5B - g\ominus10B) .
\]
If we assume that \( f \) and \( \eta \) are non-interfering, what is the relationship between \( P_{f:B}(k) \) and \( P_{\eta:B}(k) \)?

7. Suggest a morphological operator that extracts the “peaks” or “domes” of an image \( f \). You are only allowed to use translations and the conditional reconstruction operator \( r^*_B(f_n | f) \).