Introduction to Model Rocketry
Homework

Due by email before January 18th beginning of class

In this homework you will implement a numerical integration method for computing the maximum speed and maximum altitude of a model rocket following a vertical trajectory without oscillations. You will also compare your solutions with the approximated solutions we discussed in class. There is a total of 15 points.

You can use Matlab (recommended) or Excel (if you cannot use Matlab) for the computations. Please collect all your answers (including plots) in a single PDF file. Email your answers and your code or Excel sheet to tron@cis.jhu.edu before January 18th. Please include the words "Model Rocketry Homework" in your subject.

1 Preliminaries

Download the thrust curve function from the website (www.cis.jhu.edu/˜tron/teaching.php). This thrust curve corresponds to an Estes B4-4 motor with burn-out time $T_b = 1$s.

(1 point) Plot the thrust function.

(1 point) Sample the function on a number of equidistant points and compute the average thrust F_a.

2 Fehskens-Malewicky approximation

For the rest of the homework, assume that the mass of the rocket alone is 40 grams, the initial mass of the motor is 26 grams, and the mass of the propellent is 13 grams.

From class, the most general equations of motions that we considered are

$$\begin{aligned} m\dot{v} &= F - mg - kv^2 \\ \dot{y} &= v \end{aligned}, \tag{1}$$

where F is the thrust curve, v is the velocity, y is the altitude, m is the mass of the rocket and g is Earth's gravity acceleration. The constant k can be computed as $k = \frac{1}{2}\rho C_d A$, where ρ is the air viscosity (take $\rho = 1.25\frac{\text{kg}}{\text{m}^3}$ at sea level with 50°F), C_d is the drag coefficient (use $C_d = 0.8$) and A is the frontal area of the rocket (in squared meters). Assume that the rocket has a 25mm diameter body tube.

(1 point) Compute the initial mass of the rocket m_0, the mass at burnout m_b and the average mass during the thrust phase m_a.

(3 points) Use the Fehskens-Malewicky approximation formulas to obtain the maximum altitude $y_{max} = y_b + y_c$, where y_b is the altitude gained during the thrust phase and y_c is the altitude gained during the coasting phase. Be careful with the units. Remember that Newtons correspond to $\text{N} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$.

$$v_b = \sqrt{\frac{F_a - m_a g}{k}} \tanh\left(\frac{T_b}{m_a}\sqrt{k(F_a - m_a g)}\right) \tag{2}$$

$$y_b = \frac{m_a}{k} \log\left(\cosh\left(\frac{T_b}{m_a}\sqrt{k(F_a - m_a g)}\right)\right) \tag{3}$$

$$y_c = \frac{m_b}{2k} \log\left(\frac{k v_b^2}{m_b g} + 1\right) \tag{4}$$

3 Numerical approximation

(4 points) Use the explicit Euler's method

$$x(t+h) = x(t) + hf(t, x(t)) \tag{5}$$

or the mid-point method explained in class to compute a numerical solution to equation (1). You can use $h = 0.01$s and a simulation time of 10s. Note that in our case $x(t) = \begin{bmatrix} v(t) \\ y(t) \end{bmatrix}$.

(1 point) Plot the solutions $v(t)$, $y(t)$ and find the maximum altitude.

4 Analysis

(1 point) Compare the results from the closed-form and the numerical approximations and comment on the difference.

(3 points) Repeat the calculations above by varying one of the parameters. For instance, vary the rocket mass or dimensions, or change the thrust curve. Comment on the result.

5 Feedback

(2 points, Extra Credit) Please comment on what aspects of this homework and of the lectures you found most interesting and/or confusing.