Derivation of Eq. (1)

$$\left.\begin{aligned} T_\theta &= K\left(\frac{\Delta r}{r_0}+\frac{\Delta l}{l_0}\right)+\mu\left(\frac{\Delta r}{r_0}-\frac{\Delta l}{l_0}\right) \\ T_C &= K\left(\frac{\Delta r}{r_0}+\frac{\Delta l}{l_0}\right)-\mu\left(\frac{\Delta r}{r_0}-\frac{\Delta l}{l_0}\right) \end{aligned}\right\}\text{from Evans \& Skalak}$$

Laplace Law $T_\theta = R\Delta P \quad T_C = R\Delta P/2$ implies

$$\begin{bmatrix} K+\mu & K-\mu \\ K-\mu & K+\mu \end{bmatrix}\begin{bmatrix} \Delta r/r_0 \\ \Delta l/l_0 \end{bmatrix} = \begin{bmatrix} R\Delta P \\ R\Delta P/2 \end{bmatrix}$$

Or

$$\begin{bmatrix} \Delta r/r_0 \\ \Delta l/l_0 \end{bmatrix} = \frac{R\Delta P}{8K\mu}\begin{bmatrix} K+\mu & -(K-\mu) \\ -(K-\mu) & K+\mu \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{R\Delta P}{8K\mu}\begin{bmatrix} K+3\mu \\ -(K-3\mu) \end{bmatrix}$$

Derivation of Eq. (6)

Eq. (5) implies $C_i v_w = C_{i_0} v_{w_0}$

Define $\varphi = \frac{v_{w_0}}{v_0}$ (not $\frac{v_{w_0}}{v_W}$ in paper!)

Which is the osmotically active fraction of intracellular water in initial state i.e.

$$v_w(t) = v(t) - v_0(1-\varphi)$$

The first term is the volume involved in the osmotic transport; the second term is the true volume; the third term is the fraction of initial volume not involved in osmotic transport.

Thus $v_w = v - v_0(1-\varphi)$ and $v_{w_0} = v_0\varphi$ and hence $C_{i_0}\varphi v_0 = C_i\left(v - v_0(1-\varphi)\right)$

Derivation of Eq. (7)

From Eq. (6), $v - v_0(1-\varphi) = \frac{C_{i_0}\varphi v_0}{C_i} = \frac{\varphi v_0}{C_i/C_{i_0}}$

i.e.

$$\frac{v}{v_0} = \frac{\varphi}{C_i/C_{i_0}} + 1 - \varphi$$

$$j = 0 \quad \Rightarrow P_i - P_e - RTC_i + RTC_e = 0$$

$$\Rightarrow \Delta P_i + P_{i_0} - \Delta P_e - P_{e_0} - RT\Delta C_i - RTC_{i_0} + RT\Delta C_e + RTC_{e_0} = 0$$

But $P_{i_0} - P_{e_0} - RTC_{i_0} + RTC_{e_0} = 0$ and $\Delta P_e = 0$

Hence

$RT\Delta C_i = \Delta P_i + RT\Delta C_e$ i.e. $RT(C_i - C_{i_0}) = \Delta P_i + RT\Delta C_e$

$$\Rightarrow \frac{C_i}{C_{i_0}} = 1 + \frac{\Delta C_e}{C_{i_0}} + \frac{\Delta P_i}{RTC_{i_0}}$$

$$\therefore \frac{v}{v_0} = \varphi / \left[1 + \frac{\Delta C_e}{C_{i_0}} + \frac{\Delta P_i}{RTC_{i_0}} \right] + 1 - \varphi$$

Derivation of Eqs. (9) & (10)

From (6)

$$C_i - C_{i_0} = \frac{C_{i_0} \varphi v_0}{\left(v - v_0(1-\varphi)\right)} - C_{i_0} = \frac{C_{i_0}\varphi}{(v-v_0)/v_0 + \varphi} - C_{i_0} = \frac{C_{i_0}}{\frac{1}{\varphi}\frac{\Delta v}{v_0} + 1} - C_{i_0} = -\frac{C_{i_0}}{\varphi}\frac{\Delta v}{v_0} \qquad (*)$$

Eqs. (2) & (3) imply

$$\frac{dv}{dt} = -2\pi r l j = -2\pi r l L_p \left[P_i - P_e - RTC_i + RTC_e \right]$$

$$\frac{d(v_0 + \Delta v)}{dt} = -2\pi(r_0 + \Delta r)(l_0 + \Delta l) L_p \left[\Delta P_i + \left(P_{i_0} - P_{e_0} \right) - RT\Delta C_i - RTC_{i_0} + RT\Delta C_e + RTC_{e_0} \right]$$

But at $t = 0$ $P_{i_0} - P_{e_0} - RTC_{i_0} + RTC_{e_0} = 0$. So neglecting quadratic terms etc.

$$\frac{d(\Delta v)}{dt} = -2\pi r_0 l_0 L_p \left[\Delta P_i - RT\Delta C_i + RT\Delta C_e \right]$$

$$\frac{d}{dt}\left(\frac{\Delta v}{v_0}\right) = -2\frac{L_p}{r_0}\left[\frac{8K\mu}{K+9\mu}\frac{1}{r_0}\frac{\Delta v}{v_0} + RT\frac{C_{i_0}}{\varphi}\frac{\Delta v}{v_0} + RT\Delta C_e \right]$$

where the first term is from Eq. (1) and the second one from (*). Thus

$$\frac{d}{dt}\left(\frac{\Delta v}{v_0}\right) = -\frac{2L_p RT\Delta C_e}{r_0}\left[\frac{\Delta v}{v_0}\left(\frac{8K\mu}{K+9\mu}\frac{1}{r_0} + RT\frac{C_{i_0}}{\varphi} \right) / RT\Delta C_e + 1 \right] \qquad (**)$$

If $\kappa = \frac{2L_p RT\Delta C_e}{r_0}$ and $\lambda = \left(\frac{8K\mu}{K+9\mu}\frac{1}{r_0} + RT\frac{C_{i_0}}{\varphi} \right) / RT\Delta C_e$ i.e. Eq. (10)

then (**) can be written as

$$\frac{d}{dt}\left(\frac{\Delta v}{v_0}\right) = -\kappa\left[\lambda\frac{\Delta v}{v_0} + 1 \right]$$

with initial condition $\frac{\Delta v}{v_0}$ =0 at $t = 0$.

Thus

$$\frac{\Delta v}{v_0} = -\frac{1}{\lambda}\left[1 - e^{-\kappa\lambda t} \right]$$

(Eq. (9) had a typographical error)