Generative Model and consistent estimation algorithms for non-rigid deformation model

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Abstract:
The link between Bayesian and variational approaches is well known in the image analysis community in particular in the context of deformable models. However, true generative models and consistent estimation procedures are usually not available and the current trend is the computation of statistics mainly based on PCA analysis. We advocate in this paper a careful statistical modeling of deformable structures and we propose an effective and consistent estimation algorithm for the various parameters (geometric and photometric) appearing in the models.

1 Introduction
One primary difficulty in the context of deformable template models is the initial choice of the template and of various parameters in the energies underlying the registration process. This problem is of utmost importance in the context of medical imaging and computational anatomy where people try to provide statistical models for anatomical and functional variability, but also in many problems of object detection and scene interpretation. Building real generative model, that handle pose variability and yield effective likelihood ratio tests for various discriminative purposes, is a fundamental issue mainly unexplored in the context of non-rigid objects.

A first step toward a statistical approach for the estimation of templates has been proposed by C.A. Glassey and K.V. Mardia in 2001. Our goal here is to propose a coherent statistical framework for deformable templates both in terms of the probability model, and in terms of the effective estimation procedure of the template end of the deformation correspondence structure.

2 The Observation Model
Let $(y_{ij})$ be the gray level observed data. Each $y_{ij}$ is defined on a grid of size $A = K^2$ where for each $i, \Delta, e_i$ is the location of point $e$ in a specified domain $D \subset \mathbb{R}^2$. The template is a function from $\mathbb{R}^2$ to $\mathbb{R}$ we consider a small deformation framework to concrete the observation; we assume the existence of an unobserved deformation $d_i: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that $y_{ij} = f(d_i(x_{ij})) = (y_{ij})_{d_i}$ where $(x_{ij})$ are i.i.d. $N(0,1)$, independent of all other variables.

3 The Template and Deformation Model
The template $f_i$ and the deformation $d_i$ belong to $V_{p_i}$ and $V_{d_i}$ a RKHS with respective kernels $K_p$ and $K_d$. Given $(y_{ij})_{i=1:K^2, j=1:N}$, the model parameters are $\theta = (\alpha, \beta, \gamma_i)_{1:K^2}$, where $V = \{\theta, \gamma_i, \beta_i, \alpha_i, \gamma_i, \beta_i, \alpha_i, \gamma_i, \beta_i, \alpha_i\}$.

4 Parameters and Likelihood

General model (includes mixtures of deformable templates):
Model parameters: $\theta = (\alpha, \beta, \gamma_i)_{1:K^2}$ where $T = \#components$.
Weight of the different mixtures: $\nu_i = \nu_i(\theta) \sum_i \nu_i$.

For each observation $x_{ij}$ we consider the pair of unobserved variables $d_i = (\Delta, \eta_i)$.

5 The Bayesian Model

The generative probabilistic model is given by:

0.42
0.42

\begin{align*}
\phi(\theta) &= \nu_i \sum_i \nu_i(\theta) \nu_i(x_{ij}\theta, \gamma_i) \nu_i(d_i; \alpha_i, \beta_i, \gamma_i) \\
\nu_i(\theta) &= \nu_i(\theta) \sum_i \nu_i(\theta) \nu_i(x_{ij}\theta, \gamma_i) \nu_i(d_i; \alpha_i, \beta_i, \gamma_i) \\
\nu_i(\theta) &= \nu_i(\theta) \sum_i \nu_i(\theta) \nu_i(x_{ij}\theta, \gamma_i) \nu_i(d_i; \alpha_i, \beta_i, \gamma_i) \\
\nu_i(\theta) &= \nu_i(\theta) \sum_i \nu_i(\theta) \nu_i(x_{ij}\theta, \gamma_i) \nu_i(d_i; \alpha_i, \beta_i, \gamma_i)
\end{align*}

6 Estimation: Theoretical results in the 1 component case

Theorem 1 (Existence of the MAP estimator)
For any sample $y_{ij}$, there exists $\theta_1 \in \Theta$ such that

$$\phi(\theta_0) = \sup \{\phi(\theta) \mid \theta \in \Theta\}$$

Theorem 2 (Consistency) Assume that $\theta_0$ is non-empty. Then, for any compact set $K \subset \Theta$ where $P(\theta_0, \Omega) \in K \rightarrow 0$.

7 Estimation with the EM algorithm

The first iteration of the EM algorithm affects each image in a class and for any $\theta_{(i)}$ we have

$$\phi(\theta_{(i)}) = \sum_i \nu_i(\theta) \nu_i(x_{ij}\theta, \gamma_i) \nu_i(d_i; \alpha_i, \beta_i, \gamma_i)$$

8 Experiments: Estimated templates

Training set: 20 images per class 1 for component 1 and 40 for 2 components.
Results after 20 EM iterations.

9 The estimated geometric distribution

To be able to notice the geometrical effects learned through the covariance matrix we use the effect of one key point on the deformation on the corresponding template and on other elements either in the same class or in an other class.

10 In the presence of noise

The stochastic process has shown more robustness and accuracy in the presence of noise. The mode approximation is biased because of the high number of local maxima of the likelihood.

11 Multi-component case in the stochastic EM algorithm: Some problems encountered

In this particular framework, the theoretical convergence of the Markov Chain cannot be numerically reached. To generate the new simulation of each missing data, we use a Gibbs sampler procedure. The first iteration of the EM algorithm affect each image in a class with probability 1/2 then to change of class occurs, the probability for an image to be affected to an other class is too small and generally under the computer precision.

Solutions currently studied:

- Model a mixture of the previous model and other missing variables: the deformations of an image and the weight of an image for each class of $(f_{ij}, t_{ij})$.
- Consider an other simulation method based on the Gibbs sampler for the deformation and on another lass for the class of a given image. (Theory and algorithm in progress).

Références